| ERNATIONAL A LEVEL                                      |               |         |  |
|---|---------------|---------|--|
| rther Pure Maths 2                                      | Solution Bank | Pearson |  |
| Chapter review 6  |               | •       |  |
| 1 a $\frac{dy}{dx} + y \tan x = e^x \cos x$             |               |         |  |
| dx  |               |         |  |
| $e^{\int dx  dx} = \sec x$                              |               |         |  |
| $\sec x \frac{dy}{dx} + y \sec x \tan x = e^x$          |               |         |  |
|   |               |         |  |
| $y \sec x = e^x + k$                                    |               |         |  |
| •   |               |         |  |
| $y = e^x \cos x + k \cos x$                             |               |         |  |
| <b>b</b> at $x = \pi, y = 1$                            |               |         |  |
| $e^{\pi}\cos\pi + k\cos\pi = 1$                         |               |         |  |
| $k = \frac{1 - e^{\pi} \cos \pi}{1 - e^{\pi} \cos \pi}$ |               |         |  |
| $\frac{1}{\cos \pi}$                                    |               |         |  |
| $k = -(1 + e^{\pi})$                                    |               |         |  |
| Therefore:  |               |         |  |
| $y = e^x \cos x - (1 + e^\pi) \cos x$                   |               |         |  |
| $=(e^{x}-e^{\pi}-1)\cos x$                              |               |         |  |

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| $2 \frac{dy}{dt} = 2 \frac{dy}{dt} = $ | Form    | atted: Not Highlight |
| $\frac{2}{dx} - 3y = \sin x$   | Field   | Code Changed         |
| $e^{-3\int dx} = e^{-3x}$  | Comr    | mented [A3]: Amended |
|  | Field   | Code Changed         |
| $e^{-3x} - 3e^{-3x}y = \sin x$   | Comr    | mented [A4]: Amended |
| $e^{-3x}y = \int e^{-3x}\sin x  dx$  | Field   | Code Changed         |
|  |         |                      |
| $\int e^{-3x} \sin x  dx = -e^{-3x} \cos x - \int 3e^{-3x} \cos x  dx$   | Field   | Code Changed         |
| $-e^{-3x}\cos x - 3\left[-e^{-3x}\sin x - (3e^{-3x}\sin x)\right]$   |         |                      |
| $=-e^{-1}\cos x - 5\left[-e^{-1}\sin x - 5e^{-1}\sin x\right]$   |         |                      |
| $= -e^{-3x}\cos x - 3e^{-3x}\sin x - 9\int e^{-3x}\sin x  dx$  |         |                      |
| $=-\frac{1}{10}e^{-3x}(3\sin x + \cos x)$  |         |                      |
| $\int e^{-3x} \sin x  dx = -e^{-3x} \cos x - 3e^{-3x} \sin x - 9 \int e^{-3x} \sin x  dx$  | Field   | Code Changed         |
| $\frac{\int e^{-3x} \sin x  dx = -\frac{1}{10} e^{-3x} \left(3 \sin x + \cos x\right)$   |         |                      |
|  |         |                      |
| $e^{-3x}y = -\frac{1}{10}e^{-3x}\left(3\sin x + \cos x\right) + A$   | Field   | Code Changed         |
| $y = -\frac{1}{10} (3\sin x + \cos x) + Ae^{3x}$   | Field   | Code Changed         |
| At x = 0, y = 0  |         |                      |
| $-\frac{1}{4} + A = 0$   | Field   | Code Changed         |
| <u>1</u>   |         | Code Changed         |
| $A = \frac{1}{10}$   | Field   | Code Changed         |
| $y = -\frac{1}{10} (3\sin x + \cos x) + \frac{1}{10} e^{3x}$   | Field   | Code Changed         |

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| 2  dy  (4  -2)  |         |         |           | Formatted: Not Highlight |
| $3 \frac{dx}{dx} = x(4-y)$  |         |         | $\leq$    | Field Code Changed       |
| $\frac{1}{1} \frac{dy}{dy} = x$   |         |         | 1         | Field Code Changed       |
| $4-y^2 dx$  |         | /       | /         |                          |
| $\int \frac{1}{4 - v^2}  \mathrm{d}y = \int x  \mathrm{d}x$   |         | /       | $\land$   | Field Code Changed       |
| $\frac{1}{1} = \frac{1}{1}$   |         | /       | Λ         | Field Code Changed       |
| $(4-y^2)(2-y)(2+y)$   |         | /       |           |                          |
| $\frac{1}{(2-y)(2+y)} = \frac{A}{2-y} + \frac{B}{2+y}$  |         |         | $\land$   | Field Code Changed       |
| $\frac{(2-y)(2-y)}{1 = A(2+y) + B(2-y)}$  |         | /       |           | Field Code Changed       |
| When $y = 2$  |         |         |           |                          |
| $4A = 1$ $A = \frac{1}{2}$  |         |         | Λ         | Field Code Changed       |
| $\frac{4}{\text{When } y = -2}$   |         | /       |           |                          |
| 4B = 1  |         |         | ſ         |                          |
| $B = \frac{1}{4}$   |         |         | $\land$   | Field Code Changed       |
| Therefore:  |         |         |           |                          |
| $\frac{1}{(2-\nu)(2+\nu)} = \frac{1}{4(2-\nu)} + \frac{1}{4(2+\nu)}$  |         |         | Λ         | Field Code Changed       |
| (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1   |         | /       |           | Field Code Changed       |
| $\int \frac{1}{4 - y^2}  \mathrm{d}y = \frac{1}{4} \int \frac{1}{2 - y}  \mathrm{d}y + \frac{1}{4} \int \frac{1}{2 + y}  \mathrm{d}y$ |         |         |           |                          |
| $\frac{1}{2} \int \frac{1}{1} dy + \frac{1}{2} \int \frac{1}{1} dy = \int x dx$   |         |         | Λ         | Field Code Changed       |
| $4^{j}2-y^{j}4^{j}2+y^{j}$  |         | /       |           |                          |
| $\int \frac{1}{2 - y} dy + \int \frac{1}{2 + y} dy = 4 \int x dx$   |         |         |           | Commented [A5]: Amended  |
| $-\ln(2-y) + \ln(2+y) - 2x^2 + c$   |         |         |           | Field Code Changed       |
| (2+x)   |         |         |           | Field Code Changed       |
| $\ln\left(\frac{2+y}{2-y}\right) = 2x^2 + c$  |         |         |           |                          |
| $\frac{2+y}{2} = e^{2x^2+c}$  |         |         | X         | Field Code Changed       |
| $2-y^{-c}$  |         |         |           |                          |
| $= e^{2x^2}e^c$   |         | ,       | /         |                          |
| $= A e^{2x^2}$  |         | /       |           |                          |
| Let $u = Ae^{2x^2}$   |         |         | $\square$ | Field Code Changed       |
| $\frac{2+y}{2-y} = u$   |         |         | Λ         | Field Code Changed       |
| $\frac{z-y}{2+y} = u(2-y)$  |         |         |           | Field Code Changed       |
| 2 + y = 2u - uy   |         |         |           | Field Code Changed       |
| uy + y = 2u - 2   |         |         |           | Field Code Changed       |
| $2\left(Ae^{2x^2}-1\right)$   |         |         | 1         | Field Code Changed       |
| $y = \frac{\sqrt{2}}{Ae^{2x^2} + 1}$  |         | /       |           |                          |

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| When $x = 0, y = 1$<br>$1 = \frac{2(A-1)}{4+1}$  |                                    |         |   | Field Code Changed                          |
| A+1 $A+1=2A-2$   |                                    |         |   | Field Code Changed                          |
| A=3  |                                    |         | ( | Field Code Changed                          |
| $y = \frac{2(3e^{2x^2} - 1)}{3e^{2x^2} + 1}$   |                                    |         | / | Field Code Changed                          |
| 4 $\frac{d^2y}{d^2y} + \frac{dy}{dy} + y = 0$  |                                    |         |   | Formatted: Not Highlight                    |
| $dx^2 dx$  |                                    |         |   | Field Code Changed                          |
| $m^{-} + m + 1 = 0$<br>1 + $\sqrt{1 - 4}$  |                                    |         |   | Field Code Changed                          |
| $m = \frac{1}{2}$ $= \frac{-1 \pm \sqrt{-3}}{2}$ $= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$                              |                                    |         |   |   |
| $y = e^{-\frac{1}{2}x} \left( A \cos\left(\frac{\sqrt{3}}{2}x\right) + B \sin\left(\frac{\sqrt{3}}{2}x\right) \right)$ | $\left(\frac{\sqrt{3}}{2}x\right)$ |         | / | Field Code Changed                          |
| $d^2 y$ $d^2 y$  |                                    |         | χ | Formatted: Not Highlight                    |
| $5 \frac{1}{dx^2} - 12 \frac{1}{dx} + 36y = 0$   |                                    |         |   | Field Code Changed                          |
| $m^2 - 12m + 36 = 0$   |                                    |         |   | Field Code Changed                          |
| (m-6)(m-6) = 0   |                                    |         |   | Field Code Changed                          |
| $m = 6$ $y = (A + Bx)e^{6x}$   |                                    |         |   | Field Code Changed                          |
| $6  \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 4 \frac{\mathrm{d}y}{\mathrm{d}x} = 0$                                      |                                    |         |   | Formatted: Not Highlight Field Code Changed |
| $m^2 - 4m = 0$   |                                    |         |   | Field Code Changed                          |
| m(m-4) = 0   |                                    |         |   | Field Code Changed                          |
| $\overline{m} = 0$ or $m = 4$  |                                    |         |   |   |
| $v = A + Be^{4x}$  |                                    |         |   | Field Code Changed                          |

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| Further Pure Mat  | hs 2 Solution Bank PF                         | Pearson |                          |
| $d^2 y + L^2 = 0$   |   | F       | Formatted: Not Highlight |
| $\int \frac{dx^2}{dx^2} + k  y = 0$                           |   | F       | ield Code Changed        |
| $m^2 + k^2 = 0$   |   | F       | ield Code Changed        |
| $m^2 = -k^2$  |   | F       | ield Code Changed        |
| $m = \pm ki$  | n ha  | F       | ield Code Changed        |
| $\frac{y = A\cos kx + D\sin kx}{dv}$                          |   | F       | ield Code Changed        |
| $\frac{dy}{dx} = -kA\sin kx + $                               | $B\cos kx$                                    | F       | ield Code Changed        |
| When $x = 0, y = 1$   | and $\frac{dy}{dx} = 1$                       | F       | ield Code Changed        |
| A = 1   |   | F       | ield Code Changed        |
| kB = 1  |   | F       | ield Code Changed        |
| $B = \frac{1}{k}$   |   |         |                          |
| $u = \cos kr + \frac{1}{\sin kr}$                             |   | F       | ield Code Changed        |
| $y = \cos kx + \frac{-\sin k}{k}$                             | A   |         |                          |
| $d^2 v dv dv$   |   | F       | ormatted: Not Highlight  |
| <b>8</b> $\frac{y}{dx^2} - 2\frac{y}{dx} + 10y =$             | - 0   | F       | ield Code Changed        |
| $m^2 - 2m + 10 = 0$   |   | F       | ield Code Changed        |
| $2 \pm \sqrt{4 - 40}$   |   | F       | ield Code Changed        |
| $m = \frac{2}{2}$   |   |         |                          |
| $2 \pm \sqrt{-36}$  |   |         |                          |
| =2  |   |         |                          |
| $=1\pm 3i$  |   |         |                          |
| $y = e^x (A \cos 3x +$  | $B\sin 3x$ )                                  |         | ield Code Changed        |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{x} (A\cos 3x)$ | $-B\sin 3x$ + $e^{x}(-3A\sin 3x + 3B\cos 3x)$ | F       | ield Code Changed        |
| dx  |   |         |                          |
| When $x = 0$ , $y = 0$  | and $\frac{dy}{dx} = 3$                       | F       | ield Code Changed        |
| A = 0   | <u>a</u> ut                                   | F       | ield Code Changed        |
| $\frac{A}{A+3B=3}$  |   |         | ield Code Changed        |
| B = 1   |   |         |                          |
| $y = e^x \sin 3x$   |   | F       | ield Code Changed        |

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| $d^2 y dy 12 y dy$  |               |   | /            | Formatted: Not Highlight |
| 9 a $\frac{y}{dx^2} - 4\frac{y}{dx} + 13y = e^{2x}$ (1)                         |               |   |              | Field Code Changed       |
| Let $v = ke^{2x}$   |               |   |              | Field Code Changed       |
| dv  |               |   |              | Field Code Changed       |
| $\frac{1}{\sqrt{dx}} = 2ke^{2x}$  |               |   |              | (                        |
| $d^2 V$ $d^2 r$   |               |   |              | Field Code Changed       |
| $\frac{dx^2}{dx^2} = 4ke^{2x}$  |               |   |              |                          |
| Substituting into (1) gives:<br>$(4hc^{2x} - 4(4hc^{2x}) + 12hc^{2x} - c^{2x})$ |               |   |              | Commonted [A6]: Amondod  |
| 4ke - 4(4ke) + 13ke = e   |               |   | $\leq$       |                          |
| $[\kappa = 1]$  |               |   |              | Field Code Changed       |
| Hence the particular integral is  | $e^{2x}$      |   | $\backslash$ | Commented [A7]: Amended  |
|   |               |   |              | Field Code Changed       |
| <b>b</b> $_{A}m^{2}-4m+13=0$  |               |   |              | Commented [A8]: Amended  |
| $\frac{4+\sqrt{(-4)^2-4(1)(13)}}{4+\sqrt{(-4)^2-4(1)(13)}}$                     |               | ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | $\nearrow$   | Field Code Changed       |
| $m = \frac{1 \pm \sqrt{(-1)^2 - 1(1)(13)^2}}{2}$                                |               |   |              | Field Code Changed       |
| $A + \sqrt{2c}^2$   |               |   | 1            | Field Code Changed       |
| $=\frac{4\pm\sqrt{-36}}{2}$   |               |   |              |                          |
| 2 + 2;  |               |   |              |                          |
| $= 2 \pm 31$<br>Therefore the complementary fu                                  | unction is:   | /                                       |              |                          |
| $v = e^{2x} \left( A \cos 3x + B \sin 3x \right)$                               |               |   |              | Field Code Changed       |
| And the general solution is:  |               |   |              |                          |
| $v = e^{2x} (A \cos 3x + B \sin 3x) + e^{2x}$                                   | ¢.            |   |              | Commented [A9]: Amended  |
| <i>y</i> <b>c</b> (1100000 + 2 01100) + 0                                       |               |   | $\leq$       | Field Code Changed       |
| $d^2 v$   |               |   |              | Formatted: Not Highlight |
| $10 \frac{d^2 y}{dx^2} - y = 4e^x$ (1)  |               |   |              | Field Code Changed       |
| Let $v = Axe^x$   |               |   |              | Field Code Changed       |
| dv  |               |   |              | Field Code Changed       |
| $\frac{dy}{dx} = Axe^{x} + Ae^{x}$  |               |   |              |                          |
| $d^2 V$   |               |   |              | Field Code Changed       |
| $\frac{dy}{dx^2} = Axe^x + 2Ae^x$   |               |   |              |                          |
| Substituting into (1) gives:  |               |   |              |                          |
| $Axe^{x} + 2Ae^{x} - Axe^{x} = 4e^{x}$  |               |   |              | Field Code Changed       |
| 2A = 4  |               |   |              |                          |
| <i>A</i> = 2  | _             |   |              |                          |
| Hence the particular integral is $2xe$  |               |   |              | Field Code Changed       |
| $\frac{m^2 - 1 = 0}{m - \pm 1}$   |               |   |              | Field Code Changed       |
| $m - \pm 1$<br>Therefore the complementary funct                                | tion is:      |   |              |                          |
| $v = Ae^{x} + Be^{-x}$  |               |   |              | Field Code Changed       |
| And the general solution is:  |               |   |              | <b>_</b>                 |
| $v = Ae^x + Be^{-x} + 2xe^x$  |               |   |              | Field Code Changed       |
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| 11 a $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 4y = 4e^{2x}$ (1)   | Field Code Changed                          |
| $m^2 - 4m + 4 = 0$  | Field Code Changed                          |
| $\sum_{m=2}^{\infty} (m-2) = 0$   | Field Code Changed                          |
| Therefore the complementary function is:<br>$y = (A + Bx)e^{2x}$  | Field Code Changed                          |
| <b>b</b> Let $y = \lambda e^{2x}$   | Field Code Changed                          |
| $\frac{dy}{dx} = 2\lambda e^{2x}$   | Field Code Changed                          |
| $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 4\lambda \mathrm{e}^{2x}$   | Field Code Changed                          |
| Substituting into (1) gives:<br>$_{4}\lambda e^{2x} - 8\lambda e^{2x} + 4\lambda e^{2x} = 4e^{2x}$  | Field Code Changed                          |
| $\sqrt{0} = 4e^{2x}$  | Field Code Changed                          |
| This is not possible, therefore $\lambda e^{2x}$ cannot be the particular integral.   | Field Code Changed                          |
| Let $y = \lambda xe$  | Field Code Changed                          |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\lambda x \mathrm{e}^{2x} + \lambda \mathrm{e}^{2x}$  | Field Code Changed                          |
| $\frac{d^2 y}{dx^2} = 4\lambda x e^{2x} + 2\lambda e^{2x} + 2\lambda e^{2x}$ $= 4\lambda x e^{2x} + 4\lambda e^{2x}$                        | Field Code Changed                          |
| Substituting into (1) gives:<br>$4\lambda x e^{2x} + 4\lambda e^{2x} - 4(2\lambda x e^{2x} + \lambda e^{2x}) + 4\lambda x e^{2x} = 4e^{2x}$ | Field Code Changed                          |
| $4\lambda x e^{2x} + 4\lambda e^{2x} - 8\lambda x e^{2x} - 4\lambda e^{2x} + 4\lambda x e^{2x} = 4e^{2x}$                                   | Field Code Changed                          |
| $\sqrt{0} = 4e^{2x}$  | Field Code Changed                          |
| This is not possible, therefore $\lambda x e^{2x}$ cannot be the particular integral.   | Field Code Changed                          |
| <b>c</b> Let $y = kx^2 e^{2x}$  | Field Code Changed                          |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = 2kx^2\mathrm{e}^{2x} + 2kx\mathrm{e}^{2x}$   | Field Code Changed                          |
| $\frac{d^2 y}{dx^2} = 4kx^2 e^{2x} + 8kx e^{2x} + 2ke^{2x}$   | Commented [A10]: Amended Field Code Changed |
| Substituting into (1) gives:  |   |
| $4kx^{2}e^{2x} + 8kxe^{2x} + 2ke^{2x} - 4(2kx^{2}e^{2x} + 2kxe^{2x}) + 4kx^{2}e^{2x} = 4e^{2x}$   | Commented [A11]: Amended                    |
| $4kx^{2}e^{2x} + 8kxe^{2x} + 2ke^{2x} - 8kx^{2}e^{2x} - 8kxe^{2x} + 4kx^{2}e^{2x} = 4e^{2x}$  | Field Code Changed                          |
|   | Commented [A12]: Amended                    |
| Comparing coefficients for constant terms:<br>2k = 4  | Field Code Changed                          |
| $\kappa - 2$<br>—Hence the particular integral is $2x^2e^{2x}$  | Commented [A13]: Amended                    |
| Aand the general solution is:   | Field Code Changed                          |
| $y = (A+Bx)e^{2x} + 2x^2e^{2x}$   | Commented [A14]: Amended                    |
| $= \left(A + Bx + 2x^2\right) e^{2x}$   | Field Code Changed                          |
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| ther Pure Maths 2  | Solution Bank              | Pearson |                         |                          |
| $d^2 v$  |                            |         |                         | Formatted: Not Highlight |
| $12 \frac{y}{dt^2} + 4y = 5\cos 3t$ (1)                                |                            |         |                         | Field Code Changed       |
| Let $y = A\cos 3t + B\sin 3t$  |                            |         |                         | Field Code Changed       |
| $\frac{\mathrm{d}y}{\mathrm{d}t} = -3A\sin 3t + 3B\cos 3t$             |                            |         |                         | Field Code Changed       |
| $\frac{d^2 y}{d^2 y} = -9A\cos 3t - 9B\sin 3t$                         |                            |         |                         | Field Code Changed       |
| $dt^2$   |                            |         |                         |                          |
| Substituting into (1) gives:<br>$-9A\cos 3t - 9B\sin 3t + 4(A\cos 3t)$ | $3t + B\sin 3t = 5\cos 3t$ |         |                         | Field Code Changed       |
| $-9A\cos 3t - 9B\sin 3t + 4A\cos 3t$                                   | $t + 4B\sin 3t = 5\cos 3t$ |         |                         | Field Code Changed       |
| $-5A\cos 3t - 5B\sin 3t = 5\cos 3t$                                    |                            |         | I                       | Field Code Changed       |
| Comparing coefficients:  |                            |         | Ċ                       |                          |
| For $\cos 3t$ :  |                            |         | G                       |                          |
| -5A = 5  |                            |         |                         | Field Code Changed       |
| A = -1<br>For sin 3 <i>t</i> :   |                            |         |                         |                          |
| -5B = 0  |                            |         |                         | Field Code Changed       |
| B = 0  |                            |         |                         |                          |
| Hence the particular integral is                                       | $-\cos 3t$                 |         |                         | Field Code Changed       |
| $m^2 + 4 = 0$  |                            |         |                         | Field Code Changed       |
| $m = \pm 2i$   |                            |         |                         |                          |
| Therefore the complementary fu   | unction is:                |         |                         |                          |
| $y = A\cos 2t + B\sin 2t$  |                            |         |                         | Field Code Changed       |
| And the general solution is:   |                            |         |                         |                          |
| $y = A\cos 2t + B\sin 2t - \cos 3t$                                    |                            |         |                         | Field Code Changed       |
| $\frac{dy}{dy} = 24\pi i \pi 24 \pm 2R \pi \pi 24 \pm 2\pi$            |                            |         |                         | Field Code Changed       |
| $\frac{dt}{dt} = -2A\sin 2t + 2B\cos 2t + 3\sin 2t$                    | in 3t                      |         |                         |                          |
| When $t = 0$ , $y = 1$ and $\frac{dy}{dt} = 2$                         |                            |         |                         | Field Code Changed       |
| dt   |                            |         |                         |                          |
| $\frac{A-1}{B} = 1$  |                            |         | $-\!\!\!\!\!\!\!\!\!\!$ | Lommentea [A15]: Amended |
| Therefore the particular solution                                      | n is:                      |         |                         | Field Code Changed       |
| $y = \cos 2t + \sin 2t - \cos 3t$                                      |                            |         |                         | Commented [A16]: Amended |
| ►  |                            |         |                         | Field Code Changed       |

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| urther Pure Maths 2 Solution Bank  | Pearson |    |                    |
|  |         |    | Formatted:         |
| 13 a $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x + e^{2x}$ (1)   |         |    | Field Code Ch      |
| $\int dx  dx$  |         |    | Field Code Chan    |
| $\frac{dy}{dy} = \frac{dy}{dx} + dy$ |         |    | Field Code Chang   |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = \mu + 2kx\mathrm{e}^{2x} + k\mathrm{e}^{2x}$  |         |    | Field Code Chang   |
| $d^2 v$  |         | /ī | Field Code Chanc   |
| $\frac{d^2 y}{dx^2} = 4kxe^{2x} + 2ke^{2x} + 2ke^{2x}$   |         |    | <u> </u>           |
| $=4kxe^{2x}+4ke^{2x}$  |         |    |                    |
| Substituting into (1) gives:   |         |    |                    |
| $4kxe^{2x} + 4ke^{2x} - 3(\mu + 2kxe^{2x} + ke^{2x}) + 2(\lambda + \mu x + kxe^{2x}) = 4x + e^{2x}$  |         |    | Field Code Change  |
| $4kxe^{2x} + 4ke^{2x} - 3\mu - 6kxe^{2x} - 3ke^{2x} + 2\lambda + 2\mu x + 2kxe^{2x} = 4x + e^{2x}$   |         |    | Field Code Change  |
| $ke^{2x} - 3\mu + 2\lambda + 2\mu x = 4x + e^{2x}$   |         |    | Field Code Changed |
| Comparing coefficients:  |         |    |                    |
| For $e^{2x}$ :   |         |    | Field Code Changed |
| k = 1  |         |    |                    |
| For x:   |         | G  |                    |
| $2\mu = 4$   |         | !  | Field Code Changed |
| $\mu = 2$  |         |    | Field Code Changed |
| For constant terms:<br>$-3\mu + 2\lambda = 0$  |         | ~  | Field Code Charged |
| $\frac{-6+2\lambda=0}{-6+2\lambda=0}$  |         |    | Field Code Changed |
| $\lambda = 3$  |         |    | Field Code Changed |
| Hence the particular integral is $3+2x+xe^{2x}$  |         |    | Field Code Changed |
| 1  |         |    | Field Code Changed |
| <b>b</b> $_{a}m^{2}-3m+2=0$  |         |    | Field Code Changed |
| $\overline{(m-1)(m-2)} = 0$  |         |    | Field Code Changed |
| m = 1 or $m = 2$   |         |    |                    |
| Therefore the complementary function is:   |         |    |                    |
| $y = Ae^{2x} + Be^{x}$   |         |    | Field Code Changed |
| And the general solution is:   |         |    |                    |
| $y = Ae^{2x} + Be^{x} + xe^{2x} + 2x + 3$  |         |    | Field Code Changed |

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| 14 a $16\frac{d^2y}{d^2y} + 8\frac{dy}{d^2y} + 5y = 5x + 23$ (1)   | Commented [A17]: Amended |
| $\int dx^2 dx = \int $ | Formatted: Not Highlight |
| Let $y = Ax + B$   | Field Code Changed       |
| $\left \frac{\mathrm{d}y}{\mathrm{d}z}-A\right $   | Commented [A18]: Amended |
|  | Field Code Changed       |
| $\left \frac{d^2y}{d^2}\right  = 0$  | Commented [A19]: Amended |
| dx <sup>-</sup><br>Substituting into (1) gives:  | Field Code Changed       |
| 8A+5(Ax+B) = 5x+23   | Commented [A20]: Amended |
| 8A + 5Ax + 5B = 5x + 23  | Field Code Changed       |
| Comparing coefficients:  | Commented [A21]: Amended |
| For x:   | Field Code Changed       |
| $SA = 3 \Longrightarrow A = 1$   | Commented [A22]: Amended |
| $\frac{A-1}{2}$  | Field Code Changed       |
| For constant terms:  | Commented [A23]: Amended |
| $8A + 5B = 23 \Longrightarrow B = 3$   | Field Code Changed       |
| B-3  | Commented [A24]: Amended |
| Hence the particular integral is $x+3$   | Field Code Changed       |
| $-16m^2 + 8m + 5 = 0$  | Commented [A25]: Amended |
| $-8 \pm \sqrt{8^2 - 4(16)(5)}$   | Field Code Changed       |
| $m = \frac{-1}{2(16)}$   | Field Code Changed       |
|  | Field Code Changed       |
| $=\frac{-8\pm\sqrt{-256}}{-256}$   | Field Code Changed       |
| 32   |                          |
| $=\frac{-8\pm161}{222}$  |                          |
| 32   |                          |
| $=-\frac{1}{4}\pm\frac{1}{2}i$   |                          |
| Therefore the complementary function is:   | /                        |
| $y = e^{-\frac{1}{4}x} \left( A\cos\left(\frac{1}{2}x\right) + B\sin\left(\frac{1}{2}x\right) \right)$   | Field Code Changed       |
|  | /                        |
| And the general solution is:<br>$\frac{1}{2}\left(\begin{array}{c}1\\1\end{array}\right)$  | Field Code Changed       |
| $y = e^{-\frac{4}{4}x} \left( A\cos\left(\frac{1}{2}x\right) + B\sin\left(\frac{1}{2}x\right) \right) + x + 3$   | Piero Code Changeo       |
| $\frac{dy}{dt} = e^{-\frac{1}{4}x} \left( -\frac{1}{4}A\sin\left(\frac{1}{-x}\right) + \frac{1}{-B}\cos\left(\frac{1}{-x}\right) \right) - \frac{1}{-e^{-\frac{1}{4}x}} \left( A\cos\left(\frac{1}{-x}\right) + B\sin\left(\frac{1}{-x}\right) \right) + 1$  | Field Code Changed       |
| dx (2 (2) 2 (2)) 4 (2) (2))  | /                        |
| When $x = 0$ , $y = 3$ and $\frac{dy}{dx} = 3$   | Field Code Changed       |
| A + 3 = 3  |                          |
| A = 0  |                          |
| $\frac{1}{2}B - \frac{1}{4}A + 1 = 3$  | Field Code Changed       |
| R = 4  |                          |
| Therefore the particular solution is:  |                          |
| 1  |                          |
|  |                          |

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| $y = 4e^{-\frac{1}{4}x}\sin\left(\frac{1}{2}x\right) + x + 3$                                      | Field Code Changed               |
|  |                                  |
| <b>b</b> As $x \to \infty$ , $4e^{\frac{1}{4}} \sin\left(\frac{1}{2}x\right) \to 0$ so $y \to x+3$ | Field Code Changed               |
| $d^2 y dy$   | Commented [A26]: Question number |
| $15 \frac{d^{2}y}{dr^{2}} - \frac{dy}{dr} - 6y = 3\sin 3x - 2\cos 3x (1)$                          | Formatted: Indent: Left: 0 cm    |
| Let $A \cos 3x + B \sin 3x$  | Field Code Changed               |
| $\frac{dy}{dt} = -34\sin 3x + 3B\cos 3x$   | Field Code Changed               |
| dx   | Field Code Changed               |
| $\frac{d^2y}{dt} = -9A\cos 3x - 9B\sin 3x$   | Field Code Changed               |
| $\frac{dx^2}{dx^2}$ Substituting into (1) gives:   |                                  |
| $-9A\cos 3x - 9B\sin 3x + 3A\sin 3x - 3B\cos 3x - 6A\cos 3x - 6B\sin 3x = 3\sin 3x - 2\cos 3x$     | Field Code Changed               |
| $-15A\cos 3x - 15B\sin 3x + 3A\sin 3x - 3B\cos 3x = 3\sin 3x - 2\cos 3x$                           | Field Code Changed               |
| $\underline{-\cos 3x(-15A-3B)} + \sin 3x(3A-15B) = 3\sin 3x - 2\cos 3x$                            | Field Code Changed               |
| Comparing coefficients:<br>For cos 3x:   |                                  |
| -15A - 3B = -2 (1)   | Formatted: Indent: Left: 0 cm    |
| For sin 3x:  | Field Code Changed               |
| 3A - 15B = 3 (2)   | Field Code Changed               |
| Adding (1) and $5 \times$ (2) gives:<br>78R - 13   |                                  |
| -76D - 15  |                                  |
| $B = -\frac{1}{6}$   | Field Code Changed               |
|  | Commented [A27]: Amended         |
| $-A=\overline{6}$  | Field Code Changed               |
| Hence the particular integral is $\frac{1}{\cos 3x} - \frac{1}{\sin 3x}$                           | Field Code Changed               |
|  |                                  |
| $m^2 - m - 6 = 0$  | Field Code Changed               |
| (m+2)(m-3)=0   | Field Code Changed               |
| m = -2 or $m = 3Therefore the complementary function is:$  |                                  |
| $v = Ae^{3x} + Be^{-2x}$   | Field Code Changed               |
| And the general solution is:   |                                  |
| $y = Ae^{3x} + Be^{-2x} + \frac{1}{6}\cos 3x - \frac{1}{6}\sin 3x$                                 | Field Code Changed               |
| If $y(x)$ remains finite as $x \to \infty$ then $A = 0$  |                                  |
| Therefore:   |                                  |
| $v = Be^{-2x} + \frac{1}{-\cos 3x} - \frac{1}{-\sin 3x}$   | Field Code Changed               |
| $\frac{6}{2} \frac{6}{6} \frac{6}{6}$<br>When $x = 0, y = 1$                                       |                                  |
| $1 = B + \frac{1}{2}$  | Commented [A28]: Amended         |
| 6  | Field Code Changed               |
| $B = \frac{5}{6}$  | Field Code Changed               |
| Therefore the particular solution is:  |                                  |
|  |                                  |

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| $y = \frac{1}{6} \left( 5e^{-2x} + \cos 3x - \sin 3x \right)$ |               |         | Field Code Changed           |
| I   |               |         | <br>Formatted: Not Highlight |

| urther Pure Maths 2Solution BankPearson16 a $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = \cos 4t$ , $t \ge 0$ (1)Field CorLet $x = A\cos 4t + B\sin 4t$ Field Cor $\frac{dx}{dt} = -4.A\sin 4t + 4B\cos 4t$ Field Cor $\frac{d^2x}{dt} = -16A\cos 4t - 16B\sin 4t$ Field Cor $\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$ Field Cor $\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t + 4B\cos 4t$ ) + $16(A\cos 4t + B\sin 4t) = \cos 4t$ Field Cor $-16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t$ Field Cor $-16A\cos 4t - 16B\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$ Field Cor $-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Cor $-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Cor $-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Cor $-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Cor $-32A \sin 4t + 32B\cos 4t = \cos 4t$ Field Cor $-32A = 0$ Field Cor $A = 0$ Field CorHence the particular integral is $\frac{1}{32}\sin 4t$ Field Cor $\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$ Field Cor $\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m + 4) = 0}$ Field Cor $\frac{m - 4}{1 + Core (m + 4)(m +$   | TERNATIONAL A LEVEL  |           |
|---|--|-----------|
| <b>16 a</b> $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = \cos 4t, t \ge 0$ (1)<br>Let $x = A \cos 4t + B \sin 4t$<br>$\frac{dx}{dt} = -4A \sin 4t + 4B \cos 4t$<br>$\frac{d^2x}{dt^2} = -16A \cos 4t - 16B \sin 4t$<br>Substituting into (1) gives:<br>$-16A \cos 4t - 16B \sin 4t + 8(-4A \sin 4t + 4B \cos 4t) + 16(A \cos 4t + B \sin 4t) = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-32A \sin 4t + 32B \cos 4t = \cos 4t$<br>Comparing coefficients:<br>For $\cos 4t$ :<br>$\frac{32B = 1}{52}$<br>For $\sin 4t$ :<br>$\frac{-32A = 0}{A = 0}$<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$\frac{1}{32} \sin 4t$<br>Field Code C<br>$\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$<br>m = -4<br>Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t}$<br>Field Code C   | urther Pure Maths 2 Solution Bank Pearso   | on        |
| <b>16a</b> $\frac{dt^2}{dt^2} + s \frac{d}{dt} + 10.1 - cos 4t$ , $t \ge 0$ (1)<br>Let $x = Acos 4t + B \sin 4t$<br>$\frac{dt}{dt} = -4A \sin 4t + 4B \cos 4t$<br>$\frac{dt^2}{dt^2} = -16A \cos 4t - 16B \sin 4t$<br>Substituting into (1) gives:<br>$-16A \cos 4t - 16B \sin 4t + 8(-4A \sin 4t + 4B \cos 4t) + 16(A \cos 4t + B \sin 4t) = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-32A \sin 4t + 32B \cos 4t = \cos 4t$<br>Comparing coefficients:<br>For $\cos 4t$ :<br>32B = 1<br>$B = \frac{1}{32}$<br>For $\sin 4t$ :<br>-32A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$m^2 + 8m + 16 = 0$<br>(m+4)(m+4) = 0<br>m = -4<br>Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{-} \sin 4t$   | $d^2x + g^2x + g^2x + 16x - \cos(4t + 50)$ (1)   | Field Coo |
| Let $x = A \cos 4t + B \sin 4t$<br>$\frac{dx}{dt} = -4A \sin 4t + 4B \cos 4t$<br>$\frac{d^2x}{dt^2} = -16A \cos 4t - 16B \sin 4t$<br>Substituting into (1) gives:<br>$-16A \cos 4t - 16B \sin 4t + 8(-4A \sin 4t + 4B \cos 4t) + 16(A \cos 4t + B \sin 4t) = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$<br>$-32A \sin 4t + 32B \cos 4t = \cos 4t$<br>Field Code Change<br>For $\cos 4t$ :<br>32B = 1<br>$B = \frac{1}{32}$<br>For $\sin 4t$ :<br>-32A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$<br>m = -4<br>Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{-} \sin 4t$<br>Field Code Change<br>Field Code Ch | $10a \frac{dt^2}{dt^2} + 6\frac{dt}{dt} + 10x = \cos 4t, t \ge 0 $ (1)   |           |
| $\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$ Field Code Change<br>$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$ Substituting into (1) gives:<br>$-16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t$ Field Code Change<br>$-16A\cos 4t - 16B\sin 4t + 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$ Field Code Change<br>$-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Code Change<br>Field Code Cha  | $\operatorname{Let}_{A} x = A\cos 4t + B\sin 4t$   | Field Coo |
| $dt$ Field Code Change $d^2x$<br>$dt^2$ $-16A\cos 4t - 16B\sin 4t$ Field Code ChangeSubstituting into (1) gives:<br>$-16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t$ Field Code Change $-16A\cos 4t - 16B\sin 4t + 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$ Field Code Change $-16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$ Field Code Change $-32A\sin 4t + 32B\cos 4t = \cos 4t$ Field Code ChangeComparing coefficients:<br>For cos 4t:<br>$32B = 1$ Field Code Change $B = \frac{1}{32}$ Field Code ChangeFor sin 4t:<br>$-32A = 0$ Field Code Change $A = 0$ Field Code ChangeHence the particular integral is $\frac{1}{32}\sin 4t$ Field Code Change $m^2 + 8m + 16 = 0$<br>$(m+4)(m+4) = 0$ Field Code Change $m = -4$ Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$ Field Code Change $x = (A+Bt)e^{-4t}$ Field Code changeField Code Change $m = -4$ Field Code ChangeField Code ChangeTherefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$ Field Code Change $x = (A+Bt)e^{-4t}$ Field Code ChangeField Code Change <td><math display="block">\frac{\mathrm{d}x}{\mathrm{d}t} = -4A\sin 4t + 4B\cos 4t</math></td> <td>Field Coo</td>  | $\frac{\mathrm{d}x}{\mathrm{d}t} = -4A\sin 4t + 4B\cos 4t$   | Field Coo |
| $\frac{d^{2}x}{dt^{2}} = -16A\cos 4t - 16B\sin 4t$ Substituting into (1) gives:<br>$-16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t$ Field Code Change<br>$-16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$ Field Code Change<br>$-32A\sin 4t + 32B\cos 4t = \cos 4t$ Comparing coefficients:<br>For cos 4t:<br>32B = 1 Field Code Change<br>$B = \frac{1}{32}$ For sin 4t:<br>$-32A = 0$ Hence the particular integral is $\frac{1}{32}\sin 4t$ Field Code Change<br>$m^{2} + 8m + 16 = 0$ Field Code Change<br>(m + 4)(m + 4) = 0 Field Code Change<br>(m + 4)(m + 4) = 0 Field Code Change<br>Field Code Ch   | $\frac{dt}{dt}$  |           |
| df'Substituting into (1) gives:<br>$-16A \cos 4t - 16B \sin 4t + 8 (-4A \sin 4t + 4B \cos 4t) + 16 (A \cos 4t + B \sin 4t) = \cos 4t$ Field Code Change $-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t$ Field Code Change $-32A \sin 4t + 32B \cos 4t = \cos 4t$ Field Code Change $-32A \sin 4t + 32B \cos 4t = \cos 4t$ Field Code ChangeComparing coefficients:<br>For cos 4t:<br>$32B = 1$ Field Code Change $B = \frac{1}{32}$ Field Code Change $B = \frac{1}{32}$ Field Code ChangeFor sin 4t:<br>$-32A = 0$ Field Code Change $A = 0$ Field Code ChangeHence the particular integral is $\frac{1}{32} \sin 4t$ Field Code Change $m^2 + 8m + 16 = 0$ Field Code Change $(m+4)(m+4) = 0$ Field Code Change $m = -4$ Field Code ChangeTherefore the complementary function is:<br>$x = (A + Bt) e^{-4t}$ Field Code ChangeAnd the general solution is:<br>$x = (A + Bt) e^{-4t} + \frac{1}{-}\sin 4t$ Field Code Change  | $\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$   | Field Coc |
| Field Code Change<br>$ \begin{array}{c} -16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t \\ -16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t \\ -16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t \\ -32A\sin 4t + 32B\cos 4t = \cos 4t \\ -32B\sin 4t + 32B\cos 4t = \cos 4t \\ \hline Field Code Change \\ Field Code Chan$  | <u>at</u><br>Substituting into (1) gives:  |           |
| $\frac{-16A \cos 4t - 16B \sin 4t - 32A \sin 4t + 32B \cos 4t + 16A \cos 4t + 16B \sin 4t = \cos 4t}{-32A \sin 4t + 32B \cos 4t = \cos 4t}$ Field Code Change<br>Field Code Change<br>$\frac{-32A \sin 4t + 32B \cos 4t = \cos 4t}{-32B \cos 4t = \cos 4t}$ Field Code Change<br>$\frac{B = \frac{1}{32}}{\frac{1}{32}}$ Field Code Change<br>For sin 4t:<br>$\frac{-32A = 0}{A = 0}$ Hence the particular integral is $\frac{1}{32} \sin 4t$ Field Code Change<br>$\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$ Field Code Change<br>$\frac{m = -4}{Therefore the complementary function is:}$ $x = (A + Bt)e^{-4t}$ Field Code Change<br>Field Code Change   | $-16A\cos 4t - 16B\sin 4t + 8(-4A\sin 4t + 4B\cos 4t) + 16(A\cos 4t + B\sin 4t) = \cos 4t$   | Field Coo |
| $\frac{-32 A \sin 4t + 32B \cos 4t = \cos 4t}{\text{Comparing coefficients:}}$ For $\cos 4t$ :<br>$\frac{32B = 1}{32}$ For $\sin 4t$ :<br>$\frac{-32A = 0}{A = 0}$ Hence the particular integral is $\frac{1}{32} \sin 4t$ $\frac{m^2 + 8m + 16 = 0}{(m + 4)(m + 4) = 0}$ Field Code Change<br>$\frac{m = -4}{1}$ Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$ Field Code Change<br>Field Code Change   | $-16A\cos 4t - 16B\sin 4t - 32A\sin 4t + 32B\cos 4t + 16A\cos 4t + 16B\sin 4t = \cos 4t$   | Field Coo |
| Comparing coefficients:<br>For $\cos 4t$ :<br>32B = 1<br>$B = \frac{1}{32}$<br>For $\sin 4t$ :<br>-32A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$m^2 + 8m + 16 = 0$<br>(m + 4)(m + 4) = 0<br>m = -4<br>Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{-}\sin 4t$<br>Field Code Change<br>Field Code Change  | $-32A\sin 4t + 32B\cos 4t = \cos 4t$   | Field Cor |
| For $\cos 4t$ :<br>32B = 1<br>$B = \frac{1}{32}$<br>For $\sin 4t$ :<br>-32A = 0<br>A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$\frac{m^2 + 8m + 16 = 0}{(m+4)(m+4) = 0}$<br>m = -4<br>Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{-}\sin 4t$<br>Field Code Change<br>Field Code Change  | Comparing coefficients:  |           |
| Field Code Change<br>$ \begin{array}{c} B = \frac{1}{32} \\ For \sin 4t; \\ -32.4 = 0 \\ Hence the particular integral is \frac{1}{32} \sin 4t  \begin{array}{c} m^2 + 8m + 16 = 0 \\ (m+4)(m+4) = 0 \\ m = -4 \\ Therefore the complementary function is: \\ x = (A+Bt)e^{-4t} \\ And the general solution is: \\ x = (A+Bt)e^{-4t} + \frac{1}{-}\sin 4t \end{array}  Field Code Change Fi$  | For cos 4t:  |           |
| $B = \frac{1}{32}$ Field Code ChangeFor sin 4t: $-32.4 = 0$ $A = 0$ Field Code ChangeHence the particular integral is $\frac{1}{32} \sin 4t$ Field Code Change $m^2 + 8m + 16 = 0$ Field Code Change $(m+4)(m+4) = 0$ Field Code Change $m = -4$ Field Code ChangeTherefore the complementary function is: $x = (A+Bt)e^{-4t}$ And the general solution is: $x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$ Field Code ChangeField Code Change   | 32B = 1  | Field Coo |
| For sin 4t:<br>-32A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$m^2 + 8m + 16 = 0$<br>(m+4)(m+4) = 0<br>m = -4<br>Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$<br>Field Code Change<br>Field Code Change  | $B = \frac{1}{22}$   | Field Coo |
| Field Code Change<br>A = 0<br>Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$m^2 + 8m + 16 = 0$<br>(m+4)(m+4) = 0<br>m = -4<br>Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$<br>Field Code Change<br>Field Code Change   | <u>32</u><br>For sin At  |           |
| $A = 0$ Hence the particular integral is $\frac{1}{32} \sin 4t$ $m^2 + 8m + 16 = 0$ $(m+4)(m+4) = 0$ $m = -4$ Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$ Field Code Change<br>Field Code Change  | -32.4 = 0  | Field Cor |
| Hence the particular integral is $\frac{1}{32} \sin 4t$<br>$\frac{m^2 + 8m + 16 = 0}{(m+4)(m+4) = 0}$<br>m = -4<br>Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$<br>Field Code Change<br>Field Code Change<br>Field Code Change<br>Field Code Change<br>Field Code Change<br>Field Code Change  | A = 0  | ried Cot  |
| Field Code Change<br>$m^2 + 8m + 16 = 0$<br>(m+4)(m+4) = 0<br>m = -4<br>Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$<br>Field Code Change<br>Field Code Change<br>Field Code Change<br>Field Code Change   | Hence the particular integral is $\frac{1}{2} \sin 4t$   | Field Coo |
| $m^2 + 8m + 16 = 0$ Field Code Change $(m+4)(m+4) = 0$ Field Code Change $m = -4$ Therefore the complementary function is: $x = (A + Bt)e^{-4t}$ Field Code ChangeAnd the general solution is:Field Code Change $x = (A + Bt)e^{-4t}$ Field Code Change   | $\frac{11}{32} = \frac{11}{32} = 11$ |           |
| (m+4)(m+4) = 0 $m = -4$ Therefore the complementary function is:<br>$x = (A+Bt)e^{-4t}$ Field Code Change<br>And the general solution is:<br>$x = (A+Bt)e^{-4t} + \frac{1}{2}\sin 4t$ Field Code Change   | $m^2 + 8m + 16 = 0$  | Field Coo |
| m = -4 Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$ Field Code Change<br>And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{\sin 4t}$ Field Code Change   | (m+4)(m+4) = 0   | Field Coo |
| Therefore the complementary function is:<br>$x = (A + Bt)e^{-4t}$<br>And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{2}\sin 4t$<br>Field Code Change  | m = -4   |           |
| $\frac{x = (A + Bt)e^{-4t}}{\text{And the general solution is:}}$ $x = (A + Bt)e^{-4t} + \frac{1}{2}\sin 4t$ Field Code Change  | Therefore the complementary function is:   |           |
| And the general solution is:<br>$x = (A + Bt)e^{-4t} + \frac{1}{1}\sin 4t$ Field Code Change  | $x = (A + Bt)e^{-4t}$  | Field Coo |
| $x = (A + Bt)e^{-4t} + \frac{1}{-1}\sin 4t$ Field Code Change   | And the general solution is:   |           |
|   | $x = (A + Bt)e^{-4t} + \frac{1}{-1}\sin 4t$  | Field Coo |

| INTERNA | TIONAL A LEVEL  |  |            |   |                    |
|---------|---|--|------------|---|--------------------|
| Furthe  | er Pure Maths 2   | Solution Bank                                    | Pearson    |   |                    |
| 16 h    | $r = (4 + Bt)e^{-4t} + \frac{1}{2}\sin 4t$  |  |            |   | Formatted: Not     |
| 100     | $x = (A + Di)C + \frac{1}{32}SIII + i$  |  |            | 4 | Field Code Chan    |
|         | $\frac{\mathrm{d}x}{\mathrm{d}t} = -4\left(A + Bt\right)\mathrm{e}^{-4t} + B\mathrm{e}^{-4t}$ | $t^{t} + \frac{1}{8}\cos 4t$                     |            | 1 | Field Code Chang   |
|         | When $t = 0$ , $x = \frac{1}{2}$ and $\frac{dy}{dt} = \frac{1}{2}$                            | = 0  |            | 1 | Field Code Chang   |
|         | 2 dt  | ~  |            | 4 | Field Code Change  |
|         | $A = \frac{1}{2}$   |  |            | 1 | Field Code Change  |
|         | $-4A + B + \frac{1}{2} = 0$   |  |            |   | Field Code Change  |
|         | 8   |  |            |   |                    |
|         | $B = \frac{15}{8}$  |  |            |   | Field Code Change  |
|         | Therefore the particular solu   | ution is:  |            |   |                    |
|         | $x = \frac{1}{2}(4+15t)e^{-4t} + \frac{1}{22}\sin 4t$   | 4 <i>t</i>                                       |            | 1 | Field Code Change  |
|         | 8 32  |  |            |   |                    |
| 0       | As $t \to \infty$ the $e^{-4t}$ dominate  | es the first term so $\frac{1}{2}(4+15t)e^{-4t}$ | 0 leaving. | 1 | Field Code Change  |
| t       |   | $\frac{1}{8}(4+15t)c$                            | o leaving. | 4 | Field Code Change  |
|         | $x = \frac{1}{22} \sin 4t$ which is an os   | scillation.                                      |            |   | Field Code Changed |



| INTERNATIONAL A LEVEL  |   |   |         |  |
|--|---|---|---------|--|
| Further Pure Maths 2   | Solution Bank   | P   | Pearson |  |
| <b>17 b</b> But $y = 1$ when $x = 1$                             |   |   |         |  |
| $\therefore  1 = A + B - \frac{3}{4} \Longrightarrow A + B = 0$  | <sup>7</sup> / <sub>4</sub> (1)                         |   |         |  |
| $\frac{dy}{dx} = -\frac{A}{x^2} - \frac{2B}{x^3} + \frac{1}{2x}$ |   |   |         |  |
| When $x = 1$ , $\frac{dy}{dx} = 1$                               |   |   |         |  |
| $\therefore  1 = -A - 2B + \frac{1}{2} \Longrightarrow A + 2A$   | $B = -\frac{1}{2}$ (2)                                  |   |         |  |
| Solve the simultaneous equa                                      | tions (1) and (2) to give $B = -\frac{9}{4}$            | and $A = 4$                                       |         |  |
| $\therefore$ The equation of the solution                        | ion curve described is $y = \frac{4}{x} - \frac{9}{4x}$ | $\frac{1}{x^2} + \frac{1}{2} \ln x - \frac{3}{4}$ |         |  |

| INTERNATIONAL A LEVEL  |         |                    |
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| Further Pure Maths 2 Solution Bank   | Pearson |                    |
| <b>18</b> $z = \sin x$ $\therefore$ $\frac{dz}{dz} = \cos x$ and $\frac{dy}{dz} = \frac{dy}{dz} \times \cos x$               | /       | Field Code Changed |
| dx = dx = dx = dz  |         |                    |
| $\therefore  \frac{d^2 y}{dx^2} = -\frac{dy}{dz}\sin x + \cos x \frac{d^2 y}{dz^2} \times \frac{dz}{dx}$                     | /       | Field Code Changed |
| $= -\frac{dy}{dz}\sin x + \cos^2 x \frac{d^2 y}{dz^2}$   |         |                    |
| $\therefore  \frac{d^2 y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = \cos^2 x e^{\sin x}  \dagger$                          | /       | Field Code Changed |
| $\Rightarrow \cos^2 x \frac{d^2 y}{dx^2} - \sin x \frac{dy}{dx} + \tan x \cos x \frac{dy}{dx} + y \cos^2 x = \cos^2 x e^z$   |         |                    |
| $\Rightarrow \frac{d^2 y}{d^2 + y} = e^z  *$   |         |                    |
|  | ]       |                    |
| The auxiliary equation is $m^{-} + 1 = 0 \implies m = \pm 1$   |         |                    |
| $\therefore$ The c.f. is $y = A \cos z + B \sin z$   |         | Field Code Changed |
| The p.i. is $y = \lambda e^z \Rightarrow \frac{dy}{dz} = \lambda e^z$ and $\frac{d^2y}{dz^2} = \lambda e^z$                  |         | Field Code Changed |
| Substitute in * to give  |         |                    |
| $2\lambda e^{z} = e^{z} \Longrightarrow \lambda = \frac{1}{2}$   |         | Field Code Changed |
| $\therefore$ The general solution of $\star$ is $y = A \cos z + B \sin z + \frac{1}{2}e^{z}$                                 |         | Field Code Changed |
| The original equation $\dagger$ has solution   |         | Field Code Changed |
| $y = A\cos(\sin x) + B\sin(\sin x) + \frac{1}{2}e^{\sin x}$  |         | Field Code Changed |
| But $y = 1$ when $x = 0$   |         | Field Code Changed |
| $\therefore  1 = A + \frac{1}{2} \Longrightarrow A = \frac{1}{2}$  |         | Field Code Changed |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x (-A\sin(\sin x)) + \cos x (B\cos(\sin x)) + \frac{1}{2}\cos x \mathrm{e}^{\sin x}$ |         | Field Code Changed |
| As $\frac{dy}{dx} = 3$ when $x = 0$  |         | Field Code Changed |
| $\therefore  3 = B + \frac{1}{2} \Longrightarrow B = \frac{5}{2}$  |         | Field Code Changed |
| $\therefore  y = \frac{1}{2}\cos(\sin x) + \frac{5}{2}\sin(\sin x) + \frac{1}{2}e^{\sin x}$                                  | /       |                    |

| NTERNATIONAL A LEVEL   |  |         |
|--|--|---------|
| urther Pure Maths 2  | Solution Bank  | Pearson |
| Challenge  |  |         |
| <b>1 a</b> Given that $z = y^2$ , and so $y =$                                     | $= z^{\frac{1}{2}}$ and $\frac{dy}{dx} = \frac{1}{2} z^{-\frac{1}{2}} \frac{dz}{dx}$ |         |
| The equation $2(1+x^2) \frac{dy}{dt} + 2$  | $2xv = \frac{1}{2}$ becomes  |         |
|  | <i>y y</i>   |         |
| $2(1+x^2) \times \frac{1}{2}z^{-\frac{1}{2}}\frac{dz}{dx} + 2xz^{\frac{1}{2}} = x$ | z <sup>-1</sup> /2   |         |
| Multiply the equation by $\frac{z^2}{1+z}$   | $\frac{1}{2}$  |         |
| Then $\frac{dz}{dx} + \frac{2x}{1+x^2}z = \frac{1}{1+x^2}$                         |  |         |
| The integrating factor is $e^{\int \frac{2\pi}{1+x}}$                              | $\int_{x^2}^{x^2} dx = e^{\ln(1+x^2)} = 1 + x^2$                                     |         |
| $\therefore (1+x^2)\frac{\mathrm{d}z}{\mathrm{d}x} + 2xz = 1$                      |  |         |
| $\therefore  \frac{\mathrm{d}}{\mathrm{d}x}[(1+x^2)z] = 1$                         |  |         |
| $\therefore \qquad (1+x^2)z = \int 1  \mathrm{d}x$ $= x + c$                       |  |         |
| $\therefore \qquad \qquad z = \frac{x+c}{(1+x^2)}$                                 | )  |         |
| As $y = z^{\frac{1}{2}}$ , $y = \sqrt{\frac{x+c}{(1+x^2)^2}}$                      |  |         |
|  |  |         |
| <b>b</b> When $x = 0, y = 2$ $\therefore 2 =$                                      | $=\sqrt{c} \Rightarrow c = 4$  |         |
|  | $\sqrt{x+4}$   |         |

| INTERNATIONAL A LEVEL  |   |         |        |                          |
|--|---|---------|--------|--------------------------|
| Further Pure Maths 2   | Solution Bank   | Pearson |        |                          |
| dy dy du   |   |         |        | Formatted: Not Highlight |
| 2 a $\frac{1}{dx} = \frac{1}{du} \frac{1}{dx}$   |   |         |        | Field Code Changed       |
| $d^2y  d(dy  du)$  |   |         |        | Commented [A29]: Amended |
| $\frac{1}{\mathrm{d}x^2} = \frac{1}{\mathrm{d}x} \left( \frac{1}{\mathrm{d}u} \frac{1}{\mathrm{d}x} \right)$   |   |         |        | Field Code Changed       |
| $dy d^2 u = du (d^2 y du)$   |   |         |        |                          |
| $= \frac{dy}{du}\frac{du}{dx^2} + \frac{du}{dx}\left(\frac{dy}{du^2}\frac{du}{dx}\right)$  |   |         |        |                          |
| $= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 \frac{\mathrm{d}^2 y}{\mathrm{d}u^2}$  |   | /       |        |                          |
| Let $x = e^u$ , therefore:   |   |         |        |                          |
| $u = \ln x$  |   |         | C      |                          |
| $\frac{\mathrm{d}u}{\mathrm{d}u} = \frac{1}{\mathrm{d}u}$  |   |         |        | Field Code Changed       |
| dx  x  | 1   |         |        |                          |
| $= e^{u}$  |   | /       | ,      |                          |
| $\frac{d^2 u}{d^2 u^2} = -x^{-2}$  |   |         |        | Field Code Changed       |
| $dx^2$   |   |         |        |                          |
| $= -e^{2\pi}$  |   | /       | /      |                          |
| $d^2 y$ $dy$   |   |         | £      | Field Code Changed       |
| $x^2 \frac{dy}{dx^2} + 4x \frac{dy}{dx} + 2y = \ln x$  |   |         |        |                          |
| The transformed equation is:   |   | /       |        |                          |
| $\int dy (dx) = d^2y$  | [dv]]   |         | T      | Field Code Changed       |
| $e^{2u} \left[ \frac{dy}{du} \left( -e^{-2u} \right) + e^{-2u} \frac{d'y}{du^2} \right] + e^{-2u} \frac{d'y}{du^2} = e^{-2u} \left[ \frac{d'y}{du^2} \right] + e^{-2u} \frac{d'y}{du^2} = e^{-2u} \left[ \frac{d'y}{du^2} \right] + e^{-2u} \frac{d'y}{du^2} = e^{-2u} \left[ \frac{d'y}{du^2} \right] + e^{-2u} \frac{d'y}{du^2} = e^{-2u} $  | $4e^{u}\left\lfloor\frac{dy}{du}e^{-u}\right\rfloor+2y=u$ | /       |        |                          |
| $-\frac{\mathrm{d}y}{\mathrm{d}y} + \frac{\mathrm{d}^2y}{\mathrm{d}y} + 4\frac{\mathrm{d}y}{\mathrm{d}y} + 2y = u$   |   |         |        | Field Code Changed       |
| $du du^2 du^2$   |   | /       | /      |                          |
| $\frac{\mathrm{d}^2 y}{\mathrm{d} y} + 3\frac{\mathrm{d} y}{\mathrm{d} y} + 2y = y \qquad (1)$   |   |         |        | Field Code Changed       |
| $\frac{du^2}{du} \frac{du^2}{du} \frac{du}{du} $ |   | /       | /      |                          |
| Let $y = Au + B$   |   | <       |        | Commented [A30]: Amended |
| $\left[\frac{\mathrm{d}y}{\mathrm{d}x}=A\right]$   |   |         |        | Field Code Changed       |
| du   |   |         |        | Commented [A31]: Amended |
| $\left \frac{d^2y}{d^2}\right  = 0$  |   |         |        | Field Code Changed       |
| $du^2$   |   |         | $\sim$ | Commented [A32]: Amended |
| Substituting into (1) gives:<br>3A+2(Au+B) = u   |   |         |        | Field Code Changed       |
|  |   |         | (      | Field Code Changed       |
| Comparing coefficients:  |   |         |        |                          |
| For <i>u</i> :   |   |         |        | Commented [A33]: Amended |
| 2A=1   |   |         |        | Field Code Changed       |
| $A = \frac{1}{2}$  |   |         |        | Commented [A34]: Amended |
|  |   |         |        | Field Code Changed       |
| For constant terms:  |   |         |        | Commented [A35]: Amended |
| 3A + 2B = 0  |   |         |        | Field Code Changed       |
| $\frac{3}{2} + 2B = 0$   |   |         |        | Commented [A36]: Amended |
| 2  |   |         |        | Field Code Changed       |
| $B = -\frac{3}{4}$   |   |         |        | Commented [A37]: Amended |
| 4  |   | /       |        | Field Code Changed       |
| I I  |   |         | C      | -                        |

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| INTERNATIONAL A LEVEL  |                                   |         |       |                                  |  |
|--|-----------------------------------|---------|-------|----------------------------------|--|
| Further Pure Maths 2   | Solution Bank                     | Pearson |       |                                  |  |
| Hence the particular integra   | 1 is $\frac{1}{2}u - \frac{3}{4}$ |         | Field | d Code Changed                   |  |
| $\frac{d^2 y}{du^2} + 3\frac{dy}{du} + 2y = u$   | <u> </u>                          |         | Field | d Code Changed                   |  |
| $\frac{1}{m^2 + 3m + 2} = 0$ $(m+1)(m+2) = 0$  |                                   |         | Field | d Code Changed                   |  |
| m = -1  or  m = -2   | ry function is:                   |         | Field |                                  |  |
| $y = Ae^{-u} + Be^{-2u}$   |                                   |         | Field | d Code Changed                   |  |
| $y = Ae^{-u} + Be^{-2u} + \frac{1}{2}u - \frac{3}{4}$  |                                   |         | Field | d Code Changed                   |  |
| Therefore:<br>$y = \frac{A}{x} + \frac{B}{x^2} + \frac{1}{2} \ln x - \frac{3}{4}$                          |                                   |         | Field | d Code Changed                   |  |
| <b>2 b</b> $y = \frac{A}{A} + \frac{B}{A} + \frac{1}{2} \ln x - \frac{3}{2}$                               |                                   |         | Forn  | natted: Not Highlight            |  |
| $\frac{dy}{dt} = -\frac{A}{2} - \frac{2B}{2} + \frac{1}{2}$  |                                   |         | Field | d Code Changed<br>d Code Changed |  |
| $\frac{dx}{dx} = \frac{x^2}{x^3} = \frac{2x}{2x}$ When $x = 1$ , $y = 1$ and $\frac{dy}{1} = \frac{1}{2x}$ | :1                                |         | Field | d Code Changed                   |  |
| $A + B - \frac{3}{4} = 1$  |                                   |         | Field | d Code Changed                   |  |
| $A + B = \frac{7}{4}$ (2)  |                                   |         | Field | d Code Changed                   |  |
| $-A - 2B + \frac{1}{2} = 1$  |                                   |         | Field | d Code Changed                   |  |
| $-A - 2B = \frac{1}{2}$ (3)  |                                   |         | Field | d Code Changed                   |  |
| Adding (2) and (3) gives:<br>$B = -\frac{9}{2}$  |                                   |         | Field | d Code Changed                   |  |
| $\frac{4}{A+B=\frac{7}{4}}$  |                                   |         | Field | d Code Changed                   |  |
| $\frac{4}{A - \frac{9}{A} = \frac{7}{A}}$  |                                   |         | Field | d Code Changed                   |  |
| A = 4<br>Therefore the particular solution   | ution is:                         |         |       |                                  |  |
| $y = \frac{4}{x} + \frac{9}{4x^2} + \frac{1}{2}\ln x - \frac{3}{4}$  |                                   |         | Field | d Code Changed                   |  |
| 3 Substitute $u = \frac{dy}{dx}$ so equation   | becomes                           |         | Field | d Code Changed                   |  |
|  |                                   |         |       |                                  |  |

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| INTERNATIONAL A LEVEL         Further Pure Maths 2       Solution Bank $\frac{du}{dx} = u^2$ $\Rightarrow \int \frac{du}{u^2} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$ | Anternational A Level       Solution Bank       Pearson $\frac{du}{dx} = u^2$ $\Rightarrow \int \frac{du}{u^2} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{x + B}$ |  |               |         |  |
|---|---|--|---------------|---------|--|
| Further Pure Maths 2 Solution Bank<br>$\frac{du}{dx} = u^2$<br>$\Rightarrow \int \frac{du}{u^2} = \int dx$<br>$\Rightarrow -\frac{1}{u} = x + B$                            | Further Pure Maths 2 Solution Bank<br>$\frac{du}{dx} = u^2$<br>$\Rightarrow \int \frac{du}{u^2} = \int dx$<br>$\Rightarrow -\frac{1}{u} = x + B$<br>$\Rightarrow \frac{dy}{dx} = -\frac{1}{x + B}$          | TERNATIONAL A LEVEL  |               |         |  |
| $\frac{du}{dx} = u^{2}$ $\Rightarrow \int \frac{du}{u^{2}} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$  | $\frac{du}{dx} = u^{2}$ $\Rightarrow \int \frac{du}{u^{2}} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{x + B}$   | urther Pure Maths 2  | Solution Bank | Pearson |  |
| $\Rightarrow \int \frac{1}{u^2} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$   | $\Rightarrow \int \frac{dx}{u^2} = \int dx$ $\Rightarrow -\frac{1}{u} = x + B$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{x + B}$   | $\frac{\mathrm{d}u}{\mathrm{d}x} = u^2$  |               |         |  |
|   | $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{x+B}$  | $\Rightarrow \int \frac{\mathrm{d}x}{u^2} = \int \mathrm{d}x$ $\Rightarrow -\frac{1}{u} = x + B$ |               |         |  |